Closing Wed: HW_1A, 1B, 1C

Entry Task (You do): Approx. the area under $f(x) = x^3$ from x = 0 to x = 1 using n = 4 and *right-endpoints*.

Step 1: $\Delta x = \frac{b-a}{n} =$

Step 2:

$$x_0 = a =$$
$$x_1 = a + \Delta x =$$
$$x_2 = a + 2\Delta x =$$
$$x_3 = a + 3\Delta x =$$
$$x_4 = a + 4\Delta x =$$

Step 3: Plug in right-endpoints to function to get rect. heights, then add up areas (height times width).

$$Area \approx \sum_{i=1}^{4} f(x_i)\Delta x =$$

$$f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + f(x_4)\Delta x$$

I did this example again with 100 subdivisions, then 1000, then 10000. Here is a summary of my findings:

n	R_n	L_n
4	0.390625	0.140625
5	0.36	0.16
10	0.3025	0.2025
100	0.255025	0.245025
1000	0.25050025	0.24950025
10000	0.2499500025	0.2500500025

Pattern:

$$\Delta x = \frac{1-0}{n} = \frac{1}{n}, \quad x_i = 0 + i\frac{1}{n} = \frac{i}{n}$$

Adding up the area of each rectangle

$$\text{Sum} = \sum_{i=1}^{n} x_i^3 \Delta x = \sum_{i=1}^{n} \left(\frac{i}{n}\right)^3 \frac{1}{n}$$

$$\text{Area} = 0.25 = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^3 \frac{1}{n}$$

Example: Approximate the area under $f(x) = 1 + x^2$ from $x = 2$ to $x = 3$ using Riemann sums with $n = 4$ and right endpoints.

What is the general pattern in terms of *n*?

$\Delta x =$

$x_i =$

$$\sum_{i=1}^{n} f(x_i)\Delta x =$$

Another Example:

Using sigma notation, write down the general Riemann sum definition of the area from x = 5 to x = 7 under

$$f(x) = 3x + \sqrt{x}$$

$$\Delta x = \frac{b - a}{n} =$$

$$x_i = a + i\,\Delta x$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x =$$

Velocity/Distance & Reimann Sums

When velocity is a ***constant***:

Distance = Velocity · Time

Example:

You are accelerating in a car. You get the following measurements:

t (sec)	0	0.5	1.0	1.5	2.0
v(t) (ft/s)	0	6.2	10.8	14.9	18.1

Estimate the distance traveled by the car traveled from 0 to 2 seconds.

5.2 The Definite Integral

Def'n:

We define the **definite integral of f(x) from x = a to x = b** by

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x \,,$$

where $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta x$.

Basic Integral Rules:

1. $\int_a^b c\,dx = (b-a)c$

2. $\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$

3. $\int_a^b cf(x)\,dx = c\int_a^b f(x)dx$

and

$\int_a^b f(x) + g(x)\,dx$

$= \int_a^b f(x)dx + \int_a^b g(x)\,dx$

4. $\int_b^a f(x)dx = -\int_a^b f(x)dx$

Examples:

1. $\int_4^{10} 5\,dx =$

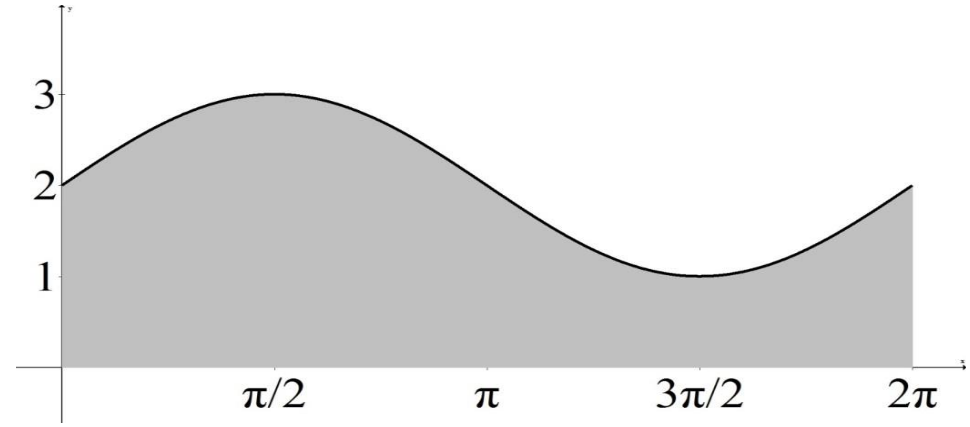
2. $\int_0^3 x^2 dx + \int_3^7 x^2 dx =$

3. $\int_0^4 5x + 3\,dx =$

4. $\int_3^1 x^3 dx = -\int_1^3 x^3\,dx$

Note on quick bounds (HW_1C: 9,10)

$$m(b-a) \le \int_a^b f(x)dx \le M(b-a)$$



Example: Consider the area under

f(x) = sin(x) + 2

on the interval *x = 0* to *x = 2π.*

(a) What is the max of f(x)? (label M)
(b) What is the min of f(x)? (label m)

(c) Draw **one** rectangle that contains all the shaded area? What can you conclude?
(d) Draw **one** rectangle that is completely inside the shaded area? Conclusion?